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In Memory of Nikolay Shcherbina

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Uniformization in complex dimension two: Domains defined by holomorphic dynamics

Eric Bedford *Stony Brook University*

Given a holomorphic dynamic system, the Fatou set is the set where the iterates form a normal family. This presents us with the problem of identifying the components of the Fatou set from the point of view of complex analysis. We will discuss this problem in complex dimension two.

A theorem of Kobayashi and its possible applications in convex geometry

Bo Berndtsson *University of Gothenburg*

If H is a Hilbert space of holomorphic functions, or sections of a holomorphic line bundle, on a complex manifold, X, we can define its Bergman kernel and associated Bergman metric. Kobayashi showed that (under mild conditions) the Ricci curvature of the Bergman metric is always smaller than $n+1$, where n is the dimension of X. I will discuss how this theorem can be applied to the study of the covariance matrix for a measure on a convex body in $\mathbb{R}^n$, and formulate an open problem about possible sharper versions of Kobayashi's theorem. (This is partly based on joint work with Mastrantonis and Rubinstein.)

Kobayashi Metric on Domains in $\mathbb{C}^2$

John Erik Fornæss *NTNU Trondheim*

This is joint work in progress with Mi Hu (Monash University, Melbourne) and Feng Rong (Jiao Tong University, Shanghai). Fatou components basically never have smooth boundary. This makes it difficult to estimate the boundary behaviour of invariant metrics. We consider the problem about Kobayashi density near the boundary of the backward orbit of a given point in a Fatou component.

The universal family of punctured Riemann surfaces is Stein

Franc Forstnerič *University of Ljubljana*

We show that the universal Teichmüller family of compact Riemann surfaces of genus $g \geq 0$ with $n > 0$ punctures is a Stein manifold. Furthermore, the space of fibrewise algebraic functions on the universal family is dense in the space of holomorphic functions, and there is a fibrewise algebraic map of the universal family to a Euclidean space which restricts to a proper embedding on every fibre. It is a challenging open problem whether every universal family can be represented as a family of proper minimal surfaces with finite total curvature in Euclidean spaces whose Weierstrass data depend holomorphically on the parameter in the corresponding Teichmüller space.

Volumes of compact Hermitian manifolds

Sławomir Kołodziej *Jagiellonian University Kraków*

Let (X, ω) be a compact Hermitian manifold of complex dimension n. The integral $\int_X \omega^n$ is the volume of X with respect to the metric ω. If we take another Hermitian metric $\omega_u = \omega + dd^c u$ (u a smooth function) then the volume does not change if ω is closed, but in general it changes.

Questions:
1. If we take the supremum of $\int_X \omega_u^n$ over u such that $\omega_u > 0$, is it finite?
2. Is the infimum strictly bigger than 0?

The answers are known in some particular cases. As shown by Guedj and Chinh ("Quasi-plurisubharmonic envelopes 2: Bounds on Monge-Amp'ere volumes"; Algebr. Geom. 9 (2022), no. 6, 688–713) those answers do not depend on ω we start with. The questions are of geometric interest, since, for instance, if the supremum is finite for given X certain statements which hold for Kähler manifolds are true also for X and remain open for general Hermitian manifolds.

On point separation

Takeo Ohsawa *Nagoya University*

Grauert's solution of the Levi problem [G] was extended by Takayama [T]. He showed that weakly 1-complete manifolds with negative canonical bundles are holomorphically convex. Recently, I arrived at the following in [Oh-1].

Theorem. *Let M be a complex manifold and let Ω be a proper bounded domain in M with C^2-smooth pseudoconvex boundary $\partial\Omega$. Assume that M admits a Kähler metric and the canonical bundle K_M of M admits a fiber metric whose curvature form is negative on a neighborhood of $\partial\Omega$. Then there exists a holomorphic map with connected fibers from Ω to $\mathbb{C}^N$ for some $N \in \mathbb{N}$ which is proper onto the image.*

Since the conclusion was weaker than the holomorphic convexity, an interest arose to decide whether or not locally Stein locally closed submanifolds of Stein manifolds are Stein, which had been asked by Griffiths in 1977 and answered by Colţoiu and Diederich in [C-D] in the 3-dimensional case. The answer was found in [Oh-2], which implies that Fornæss's counterexample to the Levi problem for ramified Riemann domains is embeddable into $\mathbb{C}^3$ as a locally closed submanifold. The point of [Oh-2] is that one can separate two points by holomorphic functions on certain complete Kähler manifolds which are not necessarily holomorphically convex. Questions and results related to this activity will be presented.

References

[C-D] Colţoiu, M and Diederich, K., *The Levi problem for Riemann domains over Stein spaces with isolated singularities,* Math. Ann. **338** (2007), 283-289.

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[Oh-1] Ohsawa, T., *On the Levi problem on Kähler manifolds under the negativity of canonical bundles on the boundary,* Pure Appl. Math. Q. **18** (2022), no. 2, 763-771.

[Oh-2] ——, *Constructing a holomorphically nonconvex surface which is locally pseudoconvex as a submanifold of* $\mathbb{C}^3$, preprint.

[T] ——, *The Levi problem and the structure theorem for non-negatively curved complete Kähler manifolds,* J. Reine Angew. Math. **504** (1998), 139-157.

Potentials of compact sets on complex manifolds (Joint work with Ragnar Sigurdsson)

Evgeny Poletsky *Syracuse University*

A potential p of a compact set K in a complex manifold X is a plurisubharmonic exhaustion function taking the least value on K and maximal outside of K. If p is unbounded, p is called a parabolic potential.
Firstly, we introduce the eccentricity $e(K, X)$ of K that takes values between 0 and 1 and show that manifolds have potentials if and only if $e(K, X) = 1$ for some K. Secondly, we show that if a parabolic potential p of a pluriregular compact set K exists, then for any other pluriregular compact set $F \subset X$ there is the unique potential q such that the function $p - q$ is bounded at infinity and Monge–Ampére masses of these functions coincide.
We also show that a parabolic manifold X either has parabolic potentials or its eccentricity is zero.

Polynomially convex embeddings of smooth manifolds

Rasul Shafikov *Western University*

An interesting problem is to determine the optimal (i.e., smallest possible) integer n so that any closed smooth real manifold of a given dimension m admits a smooth embedding into $\mathbb{C}^n$ with a polynomially convex image. It is well-known that $n > m$, and that $n = m + 1$ is already good enough for topological embeddings. However, the problem remains open for smooth embeddings. Currently, the best bound $n = m + 1$ is known for up to dimension $m \leq 11$, but for higher dimensions the bound for n remains suboptimal. In this talk I will outline the ideas and techniques involved, and indicate some difficulties in obtaining the complete solution.